Quantum-Mechanical Effect without Force for Spinning Particles.

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The concept of force is known to have only limited significance in quantum mechanics. This has been demonstrated in the Aharonov-Bohm effect, in which the particles can be viewed to move classically along trajectories unperturbed by any force, while the changes in the phase of the wave function lead to observable effects. Another thought experiment of this kind, involving neutral particles with magnetic moment, will be described below. Our motivation to analyse this example is twofold. First, experiments in which the otherwise free motion of the particles is perturbed through the quantum-mechanical phase alone provide the most striking demonstration of the fundamental difference between classical and quantum mechanics. Second, the experiment to be described is connected with the problem of the correct expression for the force acting on magnetic dipoles in classical electrodynamics.

Let us consider the classical nonrelativistic motion of a neutral magnetic dipole (*e.g.* neutron) through a long but finite plane condenser. We are to show that the centre line of the condenser is a permitted trajectory with constant velocity and magnetic moment throughout it, provided the magnetic moment is perpendicular to both the trajectory and the field.

Suppose that this is true. Let us choose the co-ordinate system with the axes x, y, z directed parallel to the field, the magnetic moment and the velocity, respectively. The magnetic field in the rest frame of the particle points in the same direction as the magnetic moment and cannot lead to any torque. To compute the force we use the formula [1]

$$\boldsymbol{F} = \operatorname{grad}(\boldsymbol{\mu}\boldsymbol{H}) - \frac{1}{c}\frac{\partial}{\partial t}[\boldsymbol{\mu}\times\boldsymbol{E}]\,, \tag{1}$$

which is valid in the rest frame. Using the Maxwell equations $\boldsymbol{F}$ can be brought into the form

$$\boldsymbol{F} = (\boldsymbol{\mu}\cdot\operatorname{grad})\boldsymbol{H} + \frac{1}{c}\left[\boldsymbol{E}\times\frac{\mathrm{d}\boldsymbol{\mu}}{\mathrm{d}t}\right]. \tag{2}$$

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(1) W. SHOCKLEY and R. P. JAMES: *Phys. Rev. Lett.*, **20**, 867 (1967); M. HUSZÁR and M. ZIEGLER-NÁRAY: *Acta Phys. Hung.*, **25**, 99 (1968); **26**, 223 (1969); P. HRASKÓ: *Nuovo Cimento*, **3** B, 213 (1971).

Since there is no torque and by assumption the velocity is constant, $\boldsymbol{\mu}$ is independent of time in the rest frame and the second term of (2) can be omitted. The magnetic field in the rest frame is proportional to $E_x = \pm |\boldsymbol{E}|$ and points in the direction of the y-axis. Therefore, only F_y can be different from zero. It is, however, proportional to $\partial E_x/\partial y$ which is zero by symmetry. Consequently, the particles move indeed freely along the centre line. It has to be emphasized, that the above argumentation holds for any point on the trajectory including those in the region of inhomogeneity of the field.

In quantum mechanics the situation is similar—except for a phase. If the free motion in the absence of the field is described by the wave function ψ_0, then in the presence of the field the wave function is modified to

$$\psi = \exp[i\theta]\psi_0 .$$

This formula follows from the quasi-classical solution of the Dirac equation [2]. The expression for θ is

$$\theta = \theta_0 - \frac{e}{2mc}\int_{\tau_0}^{\tau} \frac{\Sigma}{\Sigma_x^2 + \Sigma_y^2}(\Sigma_x, \Sigma_y, 0)\cdot \left\{\left[\boldsymbol{H} + \frac{\boldsymbol{E}\times\boldsymbol{\pi}}{mc(1+j)}\right] + \left(\frac{g}{2} - 1\right)\left[\boldsymbol{H} + \frac{\boldsymbol{E}\times\boldsymbol{\pi}}{mc} + \frac{\boldsymbol{\pi}\times(\boldsymbol{H}\times\boldsymbol{\pi})}{m^2c^2(1+j)}\right]\right\}, \tag{3}$$

where

$$\boldsymbol{\pi} = m\gamma\boldsymbol{v}, \qquad \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} .$$

When the particle has no charge only the term proportional to $g/2 - 1$ has to be retained. The charge e can be replaced by the magnetic moment according to the formula $\mu = -(e\hbar/2mc)(g/2 - 1)$, and one has to put $\Sigma_x = 0$, $\Sigma_y = \Sigma$.

It is easy to factor out the dependence of ψ on the inside region of the condenser. Inside the condenser the integration along the classical trajectory reduces to multiplication by $(\gamma v)^{-1}\cdot z$. Then the formula (3) shows that the wave function can be written in the form

$$\psi = \exp\left[i\frac{\mu E_x l}{\hbar}\right]\cdot\psi_0' , \tag{4}$$

where l is the length of the condenser.

In principle, the phase factor in (4) can be observed in the experiment shown in Fig.1. The phase differs in sign for trajectories in the upper and lower parts of the condenser. Therefore, the interference pattern on the sheet depends on the length (and on the charge Q) of the condenser, though the particles are not influenced by any force during their motion.

The complete phase of the quasi-classical wave function given in [2] is of the form $(1/\hbar)(S - \hbar\theta)$, where S is the action along the classical trajectories determined with the

(2) S. I. RUBINOW and J. B. KELLER: *Phys. Rev.*, **131**, 2789 (1963), especially formulae (66) and (68).

magnetic moment ignored. As pointed out in (2) the lack of dependence of S on the spin is the property of the ansatz rather than that of the expected solution. It is therefore natural to regard the term $-\hbar\theta$ as a correction to S for spin dependence, *i.e.* to

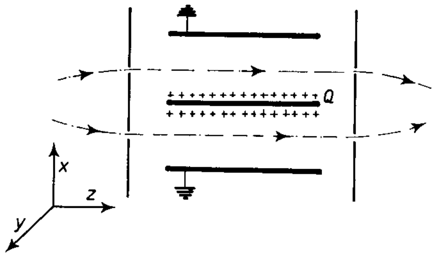

Fig. 1. – Double condenser.

identify the integrand of $-\hbar\theta$ with the interaction Lagrangian between the field and the magnetic moment (3). Then, by usual methods one finds that the force acting on the dipole is given by (1). This is an indication of the self-consistency of the argumentation.

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(3) The Lagrangian thus obtained has been discussed in another context by M. HUSZÁR: *Acta Phys. Hung.*, **23**, 225 (1967).