

Time Correlation in Tunneling of Photons

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Abstract: I propose to consider photon tunneling as a space-time correlation phenomenon between the emission and absorption of a photon on the two sides of a barrier. Standard technics based on an appropriate counting rate formula may then be applied to derive the tunneling time distribution without any *ad hoc* definition of this quantity. General formulae are worked out for a potential model using Wigner-Weisskopf method. For a homogeneous square barrier in the limit of zero tunneling probability a vanishing tunneling time is obtained.

Key words: photons, tunneling time, reduction hypothesis.

1 Introduction

The phenomenon of tunneling was recognized immediately after the birth of quantum theory as one of the most striking features of microphysics. Though tunneling probabilities are predicted unambiguously by the theory this is not the case with the time required for tunneling [1]. Tunneling time is equal to the time interval required to travel a certain distance when a barrier between the endpoints is present minus the time required to travel the distance outside the barrier. This quantity is directly measured in an experiment "with clocks" i.e. using two detectors that signalize the moment of departure and the moment of arrival of the tunneling particle (see Fig.1), but no experiment of this type has so far been actually performed. No theory is required to *infer* the value of the tunneling time from such an experiment because it is just this experiment which defines the notion "tunneling time" operationally. It may of course happen that an experiment if performed would give no sharp tunneling time but rather a tunneling time distribution.

Theory is needed to *predict* the result of the experiment. However, up to now no method has been proposed which would permit us to infer from quantum theory the expected value (or distribution) of a tunneling time experiment as described above. The Wigner-time [2], the Büttiker-Landauer time [3], [4] and the Larmor-time [5] are each based on different secondary criteria for the time spent by the particle under the barrier but neither of them has been shown to be equivalent to the primary notion of the tunneling time operationally defined by an experiment with clocks. The problem has recently been surveyed by R.Y.Chiao [6] (see also [7], [8]).

The experimental investigation of the problem became possible in the last decade thanks to the recognition that massive particles may be replaced by photons. In the experiments with photons periodic layers of alternately high

and low index media serve as optical tunnel barrier. Such experiments, while of great importance in themselves, may shed light on the tunneling of nonrelativistic massive particles as well, because the mathematical form of the equations, governing the tunneling process, are essentially the same for both cases.

The great advantage of using photons consists in the possibility of converting time intervals into phase shifts. This trick, going back to the Michelson-Morley experiment, makes the measurement of exceedingly small transit times feasible by the shift of the interference pattern in an appropriate interferometer. In the Berkeley-experiments [9], [10] UV photons were split into a pair of photons of equal frequency by spontaneous frequency down conversion in a nonlinear medium. In a Hong-Ou-Mandel spectrometer the two photons produce characteristic minimum in their rate of coincidence when pass through a beam-splitter which is in symmetrical position with respect to the beams. When, however, a thin tunneling layer was posed into the path of one of the beams the coincidence minimum could be maintained only if the beam-splitter was displaced at a certain distance. From this distance the tunneling time of the photon could be deduced and turned out to be about 2 fs while the time required to traverse the same distance in vacuum is 3.6 fs.

This result which is in rough agreement with the Wigner tunneling time indicates that the speed of tunneling may exceed the speed of light in vacuo by about 70 percent. It may be noted, however, that stationary interference experiments, contrary to the experiments with clocks, permit us to infer tunneling time only at the expense of imagining photons, moving in the arms of the equipment, as *particles*. It is this picture which might suggest the counterfactual conditional statement that if tunneling time was directly measured it would be equal to the time inferred from the stationary interference experiment. In quantum physics, however, counterfactual statements based on imagined rather than real events are in general invalid even if their validity in classical physics would be beyond doubt.

This difficulty has been clearly recognized in [6], where it was pointed out, that different experiments may lead to different tunneling times. Based on this fact the suggestion was made to abandon the unique definition of the tunneling time and replace it by a multitude of equivalent notions, assigning to each conceivable experimental setup its own value of this quantity. In particular, tunneling times measured in an experiment with clocks and deduced from a particular stationary experiment are both legitimately called tunneling time albeit under different circumstances.

But relativization of concepts often leads to confusion. It seems, therefore, safer to keep consistently to the definite meaning of tunneling time as formulated at the beginning of this section, because it conforms with the general use of the word "time", including its operational meaning. In doing so one naturally has to admit, that the time parameters inferred from experiments of other types are in general different from what is properly called tunneling time. This point of view may, perhaps, be argued for by noticing that in relativity theory only such velocities are of significance which may be related to real pairs of events through the elementary formula $\Delta s/\Delta t$ while velocities inferred from station-

ary experiments are necessarily based on imagined rather than real events. It may indeed be dangerously misleading to call "velocity" a quantity which lacks essential connotations of this term.

The purpose of the present work is to suggest a theoretical scheme to calculate the result of an experiment with clocks. The main obstacle on the way to formulate such a method is the lack of a rigorous quantum theory of the arrival time distributions of photons and particles. In quantum optics this difficulty has been overcome by the replacement of the statement "*The detector has clicked*" with the statement "*The detector atom is in one of its excited states*". Based on the assumed equivalence of these two statements (see Section 8) working formulas, known as counting rate formulas, were derived to first order in the photon-detector interaction [11] which have since been used successfully in treating space-time correlations between photons.

The direct measurement of the tunneling time of photons is actually the measurement of the time correlation between the emission and the absorption of the photon on different sides of the barrier. If, therefore, one accepts the basic assumptions underlying counting rate formulas, an appropriate formula of this kind can be derived which permits us to calculate the value (or distribution) of the tunneling time as operationally defined.

In the present work we confine ourselves (1) to the presentation of this formula and (2) to show that in the absence of any barrier it leads to the expected time correlation. In addition, it will be illustrated on a simplified model, how the theory works when a barrier is present. No attempt is made to apply the method to realistic barriers as e.g. to that used in the Berkeley-experiment.

For a very high and broad barrier our model calculation gives sharp tunneling time which is equal to zero and so it agrees *qualitatively* with the result of the Berkeley-experiment which predicts the reduction of the time required to travel a given distance when a barrier is present. This preliminary result may be an indication that the time parameter inferred from the Berkeley-experiment is indeed closely related to the tunneling time as defined in an experiment with clocks.

2 WORKING FORMULA FOR A THOUGHT EXPERIMENT WITH CLOCKS

The experimental setup is shown on Fig. 1. The photon source will be a two-level atom at the origin of the coordinate system. An atomic detector at the point $\vec{r} = (0,0,z)$ of the z coordinate axis serves to detect the photon. The barrier, an infinite homogeneous layer, supporting evanescent waves in a broad interval around the photon wavelength, is placed between the source atom and the detector perpendicular to the z -axis within the region (a,b) of this axis. An appropriate source detector is assumed to be present in the immediate vicinity of the source atom which clicks at the moment of the photon emission. It may

be assumed that it detects a particle (or another photon) which accompanies instantaneously the emission of the tunneling photon. In short, the setup works on the principle of the usual time of flight spectrometers.

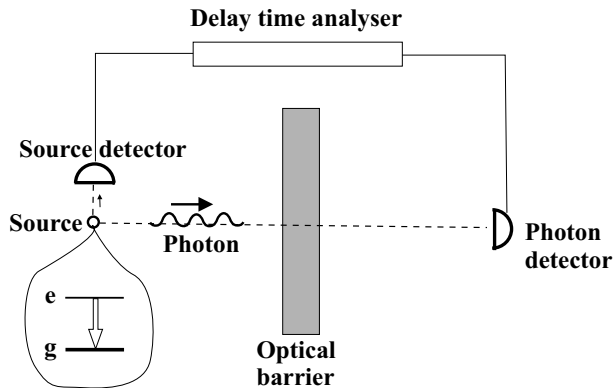


Figure 1. The setup for the time correlation measurement

Assume that at $t = 0$ the source atom and the detector atom are in their excited and ground states respectively and no photon is present. Our aim is to calculate the probability density $w(t_1, t_2)$ of the source detector clicking at t_1 and photon detector clicking at t_2 .

In order to calculate $w(t_1, t_2)$ one has first to determine the probability $p(t_1, t_2)$ of finding the source in its ground state at the moment t_1 and the photon detector in some of its excited states at t_2 . Both t_1 and t_2 are chosen arbitrarily "by ourselves" rather than by the experimental setup itself. Then, having $p(t_1, t_2)$ calculated, $w(t_1, t_2)$ is obtained by differentiation:

$$w(t_1, t_2) = \frac{\partial^2 p(t_1, t_2)}{\partial t_1 \partial t_2}. \quad (1)$$

This kind of detour through $p(t_1, t_2)$ has been used since the sixties for the calculation of photon space-time correlations in light beams prepared in various ways. In first Born-approximation the detector degrees of freedom can be easily eliminated and $p(t_1, t_2)$ and its generalizations can be expressed in terms of the expectation value of some product of field variables. Relations of this type are called counting rate formulae [11]. The restriction to first Born-approximation ensures the nonnegativity of $w(t_1, t_2)$.

This same procedure will be adapted below for the calculation of the time correlation in photon tunneling. The version of the counting rate formulae appropriate for the purposes of the present work is the following:

$$p(t_1, t_2) = \int_0^\infty d\omega \cdot \tilde{\sigma}(\omega) \cdot |M_\omega(t_1, t_2)|^2, \quad (2)$$

where $\tilde{\sigma}(\omega)$ is the spectral sensitivity of the photon detector and

$$M_\omega(t_1, t_2) = \int_0^{t_2} dt \cdot e^{i\omega t} \langle g, vac | T(\varphi^H(t, \vec{r}) P_g^H(t_1)) | e, vac \rangle. \quad (3)$$

The amplitude in the last integral is the matrix element between the states $|g, vac\rangle \equiv |g\rangle \otimes |vac\rangle$ and $|e, vac\rangle \equiv |e\rangle \otimes |vac\rangle$ of the photon vacuum and the source in its ground and excited states. P_g is the projector to the ground state of the source atom, φ is the field operator of the photon, assumed spinless, and \vec{r} is the position of the photon detector. The superscript H indicates Heisenberg picture in which the dynamical variables are driven by the Hamiltonian $H = H_s + H_f + H_{sf} \equiv H_0 + H_{sf}$ which is the sum of the source, the field and the source-field interaction Hamiltonians. The last term is assumed to be of the simple form

$$H_{sf} = Q \cdot \varphi(\vec{r} = 0). \quad (4)$$

The operator Q acts in the Hilbert space of the source, its nonzero matrix elements being $Q_{eg} = Q_{ge}^*$. The symbol T means time ordering. The main steps of the derivation of (2) and (3) are summarized in Appendix A.

3 QUANTIZATION OF THE PHOTON FIELD

The equation satisfied by the field operator φ is

$$\frac{\partial^2 \varphi}{\partial t^2} - \Delta \varphi + V \varphi = 0 \quad (5)$$

(the system of units $c = \hbar = 1$ is adopted). The last term represents the barrier, confined to the region (a, b) of the z -axis between the origin and the position of the photon detector at z . The width D of the barrier is, therefore, equal to $b - a$. For the time being no homogeneity of the barrier in the z -direction will be required.

The axial symmetry of the experimental setup suggests that quantization be performed in cylindrical coordinates:

$$\begin{aligned} \varphi(t, \vec{r}) = & \sum_{m=-\infty}^{\infty} \sum_{s=\pm} \int_0^{\infty} \frac{d\omega_r d\omega_z}{\sqrt{2\omega}} \left[a_{ms}(\omega_r, \omega_z) U_m(\omega_r | r) v_s(\omega_z | z) e^{im\varphi} e^{-i\omega t} + \right. \\ & \left. + a_{ms}^+(\omega_r, \omega_z) U_m^*(\omega_r | r) v_s^*(\omega_z | z) e^{-im\varphi} e^{i\omega t} \right], \end{aligned} \quad (6)$$

where

$$\omega = \sqrt{\omega_r^2 + \omega_z^2}, \quad (7)$$

and

$$U_m(\omega_r | r) = \frac{\sqrt{\omega_r}}{2\pi} J_m(\omega_r r). \quad (8)$$

The functions $v_{\pm}(\omega_z|z)$ obey the equation

$$-\frac{d^2 v_{\pm}}{dz^2} + V(z)v_{\pm} = \omega_z^2 v_{\pm}. \quad (9)$$

The index \pm indicates the direction of the incoming wave:

$$v_+(\omega_z|z) = \begin{cases} e^{i\omega_z(z-a)} + R e^{-i\omega_z(z-a)} & \text{if } z < a, \\ T e^{i\omega_z(z-a)} & \text{if } z > b, \end{cases} \quad (10)$$

$$v_-(\omega_z|z) = \begin{cases} T e^{-i\omega_z(z-b)} & \text{if } z < a, \\ e^{-i\omega_z(z-b)} + R' e^{i\omega_z(z-b)} & \text{if } z > b, \end{cases} \quad (11)$$

where we have used the fact that the transmission coefficient is independent of the incoming direction ($T' = T$, see Appendix B). These functions obey the relation

$$\sum_s \int d\omega_z \cdot v_s(\omega_z|z) v_s^*(\omega|z') = 2\pi \cdot \delta(z - z').$$

Equation (9) is of the form of a Schrödinger-equation for a particle moving along the z -axis ([12], [13]). Therefore, the transmission and reflection coefficients T and R are the same which are found in the textbooks on Quantum Mechanics. One has only to notice that, as the asymptotic forms (10), (11) indicate, T and R belong to a barrier of the given shape shifted to the origin (i.e. to $a = 0$, $b = D$).

4 THE WIGNER-WEISSKOPF APPROXIMATION

In the next section the matrix element in (3) will be calculated in Wigner-Weisskopf (WW) approximation [14]. This approximation scheme is based on two assumptions.

The first assumption is the confinement of the electromagnetic interaction to the subspace spanned by the vectors $|e, vac\rangle$ and $|g, 1 \text{ photon}\rangle$. The special form (4) of the interaction and the relation $J_m(0) = \delta_{m0}$ allow us to consider $m = 0$ photons only. We have, therefore, the state vector of the form

$$|t\rangle = c(t)|e, vac\rangle + \sum_{s=\pm} \int_0^{\infty} d\omega_r d\omega_z A_{\omega_r, \omega_z}^s(t) |g, \omega_r \omega_z s\rangle, \quad (12)$$

in the interaction picture, which obeys the Schrödinger-equation

$$i \frac{\partial |t\rangle}{\partial t} = H_{sf}(t) |t\rangle, \quad \left(H_{sf}(t) = e^{iH_0 t} H_{sf}(0) e^{-iH_0 t} \right), \quad (13)$$

and the the initial condition $|t = 0\rangle = |e, vac\rangle$. It follows then that the coefficients in (12) satisfy the equations

$$i\dot{c}(t) = \sum_{s=\pm} \int_0^\infty d\omega_r d\omega_z A_{\omega_r\omega_z}^s(t) e^{i(\Omega - \omega)t} \langle e, vac | H_{sf}(0) | g, \omega_r \omega_s \rangle \quad (14)$$

$$i\dot{A}_{\omega_r\omega_z}^s(t) = c(t) e^{i(\omega - \Omega)t} \langle g, \omega_r \omega_z | H_{sf}(0) | e, vac \rangle. \quad (15)$$

In these equations Ω is the excitation energy $E_e - E_g$ of the source, and ω is given by (7).

The second assumption of the WW-method consists in the Ansatz $c(t) = e^{-\Gamma t/2}$. From (15) we then have

$$\begin{aligned} A_{\omega_r\omega_z}^s(t) &= -i \int_0^t dt' \cdot e^{i(\omega - \Omega + i\Gamma/2)t'} \langle g, \omega_r \omega_s | H_{sf}(0) | e, vac \rangle = \\ &= \frac{Q_{ge}}{2\pi} \sqrt{\frac{\omega_r}{2\omega}} v_s^*(\omega_z | 0) \frac{1 - e^{i(\omega - \Omega + i\Gamma/2)t}}{\omega - \Omega + i\Gamma/2}. \end{aligned} \quad (16)$$

Now the value of Γ could be computed from (14) but since it is irrelevant for the present work we will not pursue this line any further.

We notice that the exponential Ansatz is invalid for very short times of the order of $1/\Omega$. This Ansatz prescribes the nondecay amplitude $\langle e, vac | t \rangle$ as $e^{-\Gamma t/2}$ whose time derivative at $t = 0$ is equal to $-\Gamma/2$. However, according to (13), this time derivative actually vanishes:

$$i \left[\frac{\partial \langle e, vac | t \rangle}{\partial t} \right]_{t=0} = \langle e, vac | H_{sf}(0) | e, vac \rangle = 0.$$

Owing to this shortcoming of the WW-approximation our subsequent considerations must be confined to the inside region of the future light cone of the state preparation event at $\vec{r} = 0, t = 0$. To meet this condition the relations $t_1, t_2 > z$ — or, more precisely, $(t_1 - z)\Omega \gg 1, (t_2 - z)\Omega \gg 1$ — will be assumed from the outset. This restriction is, however, irrelevant for a real experiment unless the latter is specially designed to investigate the outside region of the light cone (see Section 8).

The interaction picture of the present section is connected with the Heisenberg-picture employed in (3) by means of a unitary operator $W(t, 0)$:

$$\mathcal{O}(t) = W(t, 0) \mathcal{O}^H(t) W^\dagger(t, 0) \quad (17)$$

(notice that the absence of an upper index on operators indicates interaction picture). Then, for (12) we have

$$|t\rangle = W(t, 0) |e, vac\rangle. \quad (18)$$

Moreover, as a consequence of the first assumption the radiative corrections to $|g, vac\rangle$ must be neglected and we have the stability condition

$$W(t, 0) |g, vac\rangle = |g, vac\rangle. \quad (19)$$

5 THE FORMULA FOR $M_\omega(t_1, t_2)$ IN WW APPROXIMATION

The formula (3) for the amplitude $M_\omega(t_1, t_2)$ can be rewritten in the form

$$M_\omega(t_1, t_2) = \int_0^{t_2} dt \cdot e^{i\omega t} \langle g, vac | P_g^H(t_1) \varphi^H(t, \vec{r}) | e, vac \rangle + \theta(t_2 - t_1) \int_{t_1}^{t_2} dt \cdot e^{i\omega t} \langle g, vac | [\varphi^H(t, \vec{r}), P_g^H(t_1)] | e, vac \rangle, \quad (20)$$

where θ is the step-function.

When $t_1 > t_2$, the probability amplitude of finding the source deexcited and the photon detector excited is given by the first term of (20) alone. One may expect that this amplitude must not depend on t_1 since at the earlier moment t_2 when the photon detector was found excited the source was sure to having been already in its ground state. In WW-approximation, due to the stability condition (19), this assertion is indeed true since t_1 drops out of the integrand of the first term:

$$\begin{aligned} & \langle g, vac | P_g^H(t_1) \varphi^H(t, \vec{r}) | e, vac \rangle = \\ & = \langle g, vac | W^+(t_1, 0) P_g(t_1) W(t_1, 0) W^+(t, 0) \varphi(t, \vec{r}) W(t, 0) | e, vac \rangle = \\ & = \langle g, vac | P_g(t_1) W(t_1, t) \varphi(t, \vec{r}) | t \rangle = \langle g, vac | \varphi(t, \vec{r}) | t \rangle. \end{aligned}$$

In addition to the stability condition use was made of the fact that, in the interaction picture, $P_g(t_1)$ leaves $|g, vac\rangle$ invariant. We can, therefore, write

$$\begin{aligned} \mathcal{M}_\omega(t_2) & \equiv \int_0^{t_2} dt \cdot e^{i\omega t} \langle g, vac | P_g^H(t_1) \varphi^H(t, \vec{r}) | e, vac \rangle = \\ & = \int_0^{t_2} dt \cdot e^{i\omega t} \langle g, vac | \varphi(t, \vec{r}) | t \rangle. \end{aligned} \quad (21)$$

Using (12), the matrix element here can be cast into the form

$$\langle g, vac | \varphi(t, \vec{r}) | t \rangle = \sum_{s=\pm} \int_0^\infty d\omega_r d\omega_z A_{\omega_r \omega_z}^s(t) \langle g, vac | \varphi(t, \vec{r}) | g, \omega_r \omega_z s \rangle. \quad (22)$$

The function $A_{\omega_r \omega_z}^s(t)$ is given by (16) while the matrix element in the integrand can be calculated using (6):

$$\langle g, vac | \varphi(t, \vec{r}) | g, \omega_r \omega_z s \rangle = \frac{1}{2\pi} \sqrt{\frac{\omega_r}{2\omega}} e^{-i\omega t} v_s(\omega_z | z) \quad (23)$$

(remember that $\vec{r} = (0, 0, z)$).

The sum over s can now be performed:

$$\begin{aligned} \sum_{s=\pm} v_s^*(\omega_z|0)v_s(\omega_z|z) &= \left(e^{i\omega_z a} + R^* e^{-i\omega_z a} \right) T e^{i\omega_z(z-a)} + \\ &+ T^* e^{-i\omega_z b} \left(e^{-i\omega_z(z-b)} + R' e^{i\omega_z(z-b)} \right) = \\ &= T e^{i\omega_z z} + T^* e^{-i\omega_z z} + e^{i\omega_z z} \left(T R^* e^{-2i\omega_z a} + T^* R' e^{-2i\omega_z b} \right). \end{aligned}$$

According to (48) of the Appendix B the sum in the parentheses is equal to zero:

$$T R^* e^{-2i\omega_z a} + T^* R' e^{-2i\omega_z b} = 0, \quad (24)$$

therefore

$$\sum_{s=\pm} v_s^*(\omega_z|0)v_s(\omega_z|z) = T e^{i\omega_z z} + T^* e^{-i\omega_z z}. \quad (25)$$

Putting now (16), (23) and (25) into (22) and changing the integration variable from ω_r to $\omega = \sqrt{\omega_r^2 + \omega_z^2}$ we obtain

$$\begin{aligned} \langle g, vac | P_g^H(t_1) \varphi^H(t, \vec{r}) | e, vac \rangle &= \langle g, vac | \varphi(t, \vec{r}) | t \rangle = \\ &= \frac{Q_{ge}}{8\pi^2} \int_0^\infty d\omega \cdot \frac{e^{-i\omega t} - e^{-i(\Omega - i\Gamma/2)t}}{\omega - \Omega + i\Gamma/2} \int_0^\omega d\omega_z \left[T(\omega_z) e^{i\omega_z z} + T^*(\omega_z) e^{-i\omega_z z} \right]. \end{aligned} \quad (26)$$

The matrix element in the second integral of (20) can be handled analogously:

$$\begin{aligned} \langle g, vac | [\varphi^H(t, \vec{r}), P_g^H(t_1)] | e, vac \rangle &= \\ &= \sum_{s=\pm} \int_0^\infty d\omega_r d\omega_z [A_{\omega_r, \omega_z}^s(t_1) - A_{\omega_r, \omega_z}^s(t)] \langle g, vac | \varphi(t, \vec{r}) | g, \omega_r \omega_z s \rangle = \\ &= \frac{Q_{ge}}{8\pi^2} \left\{ e^{-i(\Omega - i\Gamma/2)t} \int_0^\infty \frac{d\omega}{\omega - \Omega + i\Gamma/2} \int_0^\omega d\omega_z \left[T(\omega_z) e^{i\omega_z z} + T^*(\omega_z) e^{-i\omega_z z} \right] - \right. \\ &\left. - e^{-i(\Omega - i\Gamma/2)t_1} \int_0^\infty \frac{d\omega}{\omega - \Omega + i\Gamma/2} \int_0^\omega d\omega_z \left[T(\omega_z) e^{i\omega_z z} + T^*(\omega_z) e^{-i\omega_z z} \right] \right\}. \end{aligned} \quad (27)$$

Here $t \geq t_1$ since this expression is the integrand in the second term of (20).

Comparing (27) with (26) we see that in WW-approximation

$$\langle g, vac | P_g^H(t_1) \varphi^H(t, \vec{r}) | e, vac \rangle = \langle g, vac | [\varphi^H(t, \vec{r}), P_g^H(0)] | e, vac \rangle. \quad (28)$$

6 TIME CORRELATION WHEN NO BARRIER IS PRESENT

In this case $T = 1$, the ω_z integral in (27) gives $2 \sin \omega z / z$ and we have

$$\langle g, vac | [\varphi^H(t, \vec{r}), P_g^H(t_1)] | e, vac \rangle^0 = \frac{Q_{ge}}{8\pi^2} [I_1^0 + I_2^0 + I_3^0 + I_4^0], \quad (29)$$

where the superscript 0 indicates the absence of the barrier and

$$\begin{aligned} I_1^0(z) &= \frac{1}{iz} e^{-i(\Omega - i\Gamma/2)t} \int_0^\infty d\omega \frac{e^{i\omega z}}{\omega - \Omega + i\Gamma/2}, \\ I_2^0(z) &= -\frac{1}{iz} e^{-i(\Omega - i\Gamma/2)t_1} \int_0^\infty d\omega \frac{e^{-i\omega(t-z-t_1)}}{\omega - \Omega + i\Gamma/2}, \\ I_3^0(z) &= -\frac{1}{iz} e^{-i(\Omega - i\Gamma/2)t} \int_0^\infty d\omega \frac{e^{-i\omega z}}{\omega - \Omega + i\Gamma/2}, \\ I_4^0(z) &= \frac{1}{iz} e^{-i(\Omega - i\Gamma/2)t_1} \int_0^\infty d\omega \frac{e^{-i\omega(t+z-t_1)}}{\omega - \Omega + i\Gamma/2}. \end{aligned} \quad (30)$$

Consider I_1^0 . The integration contour can be deformed upward to contain the positive imaginary axis and the quarter of the large circle at infinity. Since ω_z is positive, the integrand is exponentially small on this part of the large circle and gives no contribution. Moreover, the integrand is regular in the upper half plane and no pole contributions arise. We have, therefore,

$$I_1^0(z) = \frac{1}{iz} e^{-i(\Omega - i\Gamma/2)t} \int_0^\infty \frac{e^{-\eta z}}{i\eta - \Omega + i\Gamma/2} i d\eta.$$

This integral can be expanded in terms of the inverse of the large distance z , the leading term being of the order of $1/\Omega z$, and so I_1^0 turns out to be of second order. Therefore, in the leading (linear) order in $1/z$, I_1^0 must be neglected.

The same conclusion applies to the sum of I_3^0 and I_4^0 as well. In both of these terms pole contributions arise which are linear in $1/z$ but they drop out of the sum.

Consider now I_2^0 . When $t - z - t_1 > 0$ the contour has to be deformed downward and a pole term

$$I_2^0(z) = \frac{1}{iz} \theta(t - z - t_1) \cdot 2\pi i \cdot e^{-i(\Omega - i\Gamma/2)(t-z)} \quad (31)$$

arises. In the opposite case of $t - z - t_1 < 0$ the deformation is upward and no pole is to be dealt with. The contribution of the integrals along the imaginary axis is negligible only if $|t - z - t_1| \Omega \gg 1$. Therefore, $I_2^0(z)$ is given by (31) provided the step function is assumed smoothed on the scale $1/\Omega$. From an observational point of view such a smoothing is of no significance and in what follows no attention will be paid to it.

We have, therefore

$$\begin{aligned} & \langle g, vac | [\varphi^H(t, \vec{r}), P_g^H(t_1)] | e, vac \rangle^0 = \\ & = \frac{Q_{eg}}{4\pi z} \theta(t - t_1 - z) e^{-i(\Omega - i\Gamma/2)(t - z)}. \end{aligned} \quad (32)$$

From (28) we obtain

$$\begin{aligned} \mathcal{M}_\omega^0(t_2) & = - \int_0^{t_2} dt \cdot e^{i\omega t} \langle g, vac | [\varphi^H(t, \vec{r}), P_g^H(0)] | e, vac \rangle^0 = \\ & = i \frac{Q_{ge}}{4\pi z} \theta(t_2 - z) e^{i(\Omega - i\Gamma/2)z} \frac{e^{i(\omega - \Omega + i\Gamma/2)t_2} - e^{i(\omega - \Omega + i\Gamma/2)z}}{\omega - \Omega + i\Gamma/2}. \end{aligned} \quad (33)$$

The θ -function here may in fact be omitted since $t_2 > z$ by assumption.

For the integral, occurring in the second term of (20) we obtain

$$\begin{aligned} & \int_{t_1}^{t_2} dt \cdot e^{i\omega t} \langle g, vac | [\varphi^H(t, \vec{r}), P_g^H(0)] | e, vac \rangle^0 = \\ & = -i \frac{Q_{ge}}{4\pi z} \theta(t_2 - t_1 - z) e^{i(\Omega - i\Gamma/2)z} \frac{e^{i(\omega - \Omega + i\Gamma/2)t_2} - e^{i(\omega - \Omega + i\Gamma/2)(t_1 + z)}}{\omega - \Omega + i\Gamma/2}. \end{aligned} \quad (34)$$

Since $\theta(t_1) = \theta(t_2 - z) = 1$, the right hand side of (34) is equal to

$$\theta(t_2 - t_1 - z) [\mathcal{M}_\omega^0(t_1 + z) - \mathcal{M}_\omega^0(t_2)].$$

Hence, we have from (20)

$$\begin{aligned} \mathcal{M}_\omega^0(t_1, t_2) & = \mathcal{M}_\omega^0(t_2) + \theta(t_2 - t_1 - z) [\mathcal{M}_\omega^0(t_1 + z) - \mathcal{M}_\omega^0(t_2)] = \\ & = \theta(t_1 + z - t_2) \mathcal{M}_\omega^0(t_2) + \theta(t_2 - t_1 - z) \mathcal{M}_\omega^0(t_1 + z). \end{aligned} \quad (35)$$

Substituting this into (2) we obtain

$$\begin{aligned} p^0(t_1, t_2) & = \theta(t_1 + z - t_2) \int_0^\infty d\omega \cdot \tilde{\sigma}(\omega) |\mathcal{M}_\omega^0(t_2)|^2 + \\ & + \theta(t_2 - t_1 - z) \int_0^\infty d\omega \cdot \tilde{\sigma}(\omega) |\mathcal{M}_\omega^0(t_1 + z)|^2. \end{aligned}$$

In arrival time measurements the spectral sensitivity must be as broad as possible so we assume $\tilde{\sigma}(\omega) = \tilde{\sigma} = \text{constant}$. Then, substituting (33), we find

$$\int_0^\infty d\omega \cdot \tilde{\sigma}(\omega) |\mathcal{M}_\omega^0(t_2)|^2 = \frac{|Q_{eg}|^2 \tilde{\sigma}}{4\pi z^2} \cdot \frac{1}{\Gamma} \left(1 - e^{-\Gamma(t_2 - z)} \right), \quad (36)$$

which leads to

$$p^0(t_1, t_2) = \frac{|Q_{eg}|^2 \tilde{\sigma}}{4\pi z^2} \cdot \frac{1}{\Gamma} \left\{ 1 - \theta(t_1 + z - t_2) e^{-\Gamma(t_2 - z)} - \theta(t_2 - t_1 - z) e^{-\Gamma t_1} \right\}. \quad (37)$$

The formula (1) indicates that $w^0(t_1, t_2)$ may be different from zero only around $t_1 + z - t_2 = 0$. Putting

$$w^0(t_1, t_2) = W(t_2) \cdot \delta(t_2 - t_1 - z),$$

we find

$$\begin{aligned} W(t_2) &= \int_{t_2-z-\epsilon}^{t_2-z+\epsilon} dt_1 \cdot w^0(t_1, t_2) = \int_{t_2-z-\epsilon}^{t_2-z+\epsilon} dt_1 \cdot \frac{\partial^2 p^0(t_1, t_2)}{\partial t_1 \partial t_2} = \\ &= \left[\frac{\partial p^0(t_1, t_2)}{\partial t_2} \right]_{t_1=t_2-z+\epsilon} - \left[\frac{\partial p^0(t_1, t_2)}{\partial t_2} \right]_{t_1=t_2-z-\epsilon}. \end{aligned}$$

By (37) the second term is zero while in the limit of $\epsilon = 0$ the first one is equal to $\frac{|Q_{ge}|^2 \tilde{\sigma}}{4\pi z^2} \cdot e^{-\Gamma(t_2 - z)}$. Hence finally

$$w^0(t_1, t_2) = \frac{|Q_{ge}|^2 \tilde{\sigma}}{4\pi z^2} \cdot e^{-\Gamma t_1} \cdot \delta(t_2 - t_1 - z). \quad (38)$$

Though this is just the expected result it is far from being an obvious consequence of the counting rate formula (2), (3) and of the reasoning in Appendix A which led to it.

7 TIME CORRELATION IN THE PRESENCE OF A RECTANGULAR BARRIER

Let us choose in (9) a constant $V(z)$ equal to μ in the interval (a, b) of width D and zero outside. Since (9) is, up to constant coefficients, identical to the non-relativistic Schrödinger-equation, the transmission coefficient of this rectangular barrier can be taken over from quantum mechanics:

$$T(\omega_z) = e^{-i\omega_z D} \cdot e^{-D\sqrt{\mu^2 - \omega_z^2}} \cdot \frac{1 - e^{4i\alpha(\omega_z)}}{1 - e^{4i\alpha(\omega_z)} \cdot e^{-2D\sqrt{\mu^2 - \omega_z^2}}}, \quad (39)$$

in which

$$e^{2i\alpha(\omega_z)} = \frac{\omega_z - i\sqrt{\mu^2 - \omega_z^2}}{\omega_z + i\sqrt{\mu^2 - \omega_z^2}}.$$

The function $T(\omega_z)$ has a cut along $\omega_z > \mu$. The physical values are those on the upper edge of the cut where $\sqrt{\mu^2 - \omega_z^2} = -i\sqrt{\omega_z^2 - \mu^2}$.

The right hand side of (27) can now be written as the sum

$$\langle g, vac | [\varphi^H(t, \vec{r}), P_g^H(t_1)] | e, vac \rangle = \frac{Q_{ge}}{8\pi^2} [I_1 + I_2 + I_3 + I_4],$$

where

$$\begin{aligned}
I_1(z) &= e^{-i(\Omega - i\Gamma/2)t} \int_0^\infty \frac{d\omega}{\omega - \Omega + i\Gamma/2} \int_0^\omega d\omega_z \cdot T(\omega_z) e^{i\omega_z z}, \\
I_2(z) &= -e^{-i(\Omega - i\Gamma/2)t_1} \int_0^\infty d\omega \frac{e^{-i\omega(t-t_1)}}{\omega - \Omega + i\Gamma/2} \int_0^\omega d\omega_z \cdot T(\omega_z) e^{i\omega_z z}, \\
I_3(z) &= e^{-i(\Omega - i\Gamma/2)t} \int_0^\infty \frac{d\omega}{\omega - \Omega + i\Gamma/2} \int_0^\omega d\omega_z \cdot T^*(\omega_z) e^{-i\omega_z z}, \\
I_4(z) &= -e^{-i(\Omega - i\Gamma/2)t_1} \int_0^\infty d\omega \frac{e^{-i\omega(t-t_1)}}{\omega - \Omega + i\Gamma/2} \int_0^\omega d\omega_z \cdot T^*(\omega_z) e^{-i\omega_z z}.
\end{aligned} \tag{40}$$

We will argue below that the formula (38), derived in the absence of a barrier, remains to reasonable accuracy valid also for a sufficiently high and broad rectangular barrier provided z is replaced in it by $z - D$.

The variable z in the formulae (40) appears only in the inner integrals. If the first exponential factor of T is separated: $T(\omega_z) = e^{-i\omega_z D} \mathcal{T}(\omega_z)$, then the exponentials in these integrals become $e^{\pm i\omega_z(z - D)}$.

The essential contribution to the I_j -s must come from the region of integration around Ω . If $\mu \gg \Omega$ the exponent $e^{-2D\sqrt{\mu^2 - \omega_z^2}}$ in the denominator is very small in this region. We expand the fraction in $T(\omega_z)$ in terms of this small quantity and retain from the resulting asymptotic expansion the first term only (i.e. we disregard the second term of the denominator). Then,

$$\mathcal{T}(\omega_z) = e^{-D\sqrt{\mu^2 - \omega_z^2}} \left[1 - \left(\frac{\omega_z - i\sqrt{\mu^2 - \omega_z^2}}{\mu} \right)^4 \right]. \tag{41}$$

Consider the behaviour of the factor $e^{-D\sqrt{\mu^2 - \omega_z^2}}$ when $|\omega_z| \rightarrow \infty$ along some direction φ on the upper half ω_z -plane. Then the value $\varphi = 0$ corresponds to the upper edge of the cut and approaching infinity we have $\sqrt{\mu^2 - \omega^2} \sim -i|\omega_z|$. Therefore, for positiv φ

$$\sqrt{\mu^2 - \omega_z^2} \sim -i|\omega_z| e^{i\varphi} = -i|\omega_z| \cdot \cos \varphi + |\omega_z| \cdot \sin \varphi,$$

so that $\left| e^{-D\sqrt{\mu^2 - \omega_z^2}} \right|$ approaches zero exponentially when $|\omega_z| \rightarrow \infty$.

Consider the inner integral

$$F(\omega) \equiv \int_0^\omega d\omega_z \cdot \mathcal{T}(\omega_z) e^{i\omega_z(z - D)}$$

in (40). Since its integrand vanishes exponentially on the large circle in the upper half plane the integration contour can be deformed in this direction into

two semiinfinite straight lines along the positive imaginary direction:

$$F(\omega) = i \int_0^\infty d\eta \cdot \mathcal{T}(i\eta) \cdot e^{-\eta(z-D)} - \\ -ie^{i\omega(z-D)} \int_0^\infty d\eta \cdot \mathcal{T}(\omega + i\eta) \cdot e^{-\eta(z-D)},$$

which, to first order in $1/(z-D)$ is equal to

$$F(\omega) = \frac{i}{z-D} [\mathcal{T}(0) - e^{i\omega(z-D)} \mathcal{T}(\omega)].$$

But $\mathcal{T}(0) = 0$, hence

$$F(\omega) = \frac{e^{i\omega(z-D)}}{i(z-D)} \mathcal{T}(\omega).$$

Substituting this into (40) we obtain

$$I_1(z) = \frac{1}{i(z-D)} e^{-i(\Omega - i\Gamma/2)t} \int_0^\infty d\omega \frac{\mathcal{T}(\omega) e^{i\omega(z-D)}}{\omega - \Omega + i\Gamma/2}, \\ I_2(z) = -\frac{1}{i(z-D)} e^{-i(\Omega - i\Gamma/2)t_1} \int_0^\infty d\omega \frac{\mathcal{T}(\omega) e^{-i\omega(t-z+D-t_1)}}{\omega - \Omega + i\Gamma/2}, \\ I_3(z) = -\frac{1}{i(z-D)} e^{-i(\Omega - i\Gamma/2)t} \int_0^\infty d\omega \frac{\mathcal{T}^*(\omega) e^{-i\omega(z-D)}}{\omega - \Omega + i\Gamma/2}, \\ I_4(z) = \frac{1}{i(z-D)} e^{-i(\Omega - i\Gamma/2)t_1} \int_0^\infty d\omega \frac{\mathcal{T}^*(\omega) e^{-i\omega(t+z-D-t_1)}}{\omega - \Omega + i\Gamma/2}.$$

The direction of deformation of the contours in these integrals remain the same as it was in (30) since the factor $e^{-D\sqrt{\mu^2 - \omega_z^2}}$ in $\mathcal{T}(\omega)$ leaves the behaviour of the integrand at infinity unchanged. We may, therefore, write for the first two line of (40)

$$I_j(z) = \mathcal{T}(\Omega - i\Gamma/2) \cdot I_j^0(z-D)$$

and an analogous expression with \mathcal{T}^* for the remaining lines. Hence

$$w(t_1, t_2) = \frac{|Q_{ge} \cdot \mathcal{T}(\Omega - i\Gamma/2)|^2 \tilde{\sigma}}{4\pi(z-D)^2} \cdot e^{-\Gamma t_1} \cdot \delta(t_2 - t_1 - z + D). \quad (42)$$

Since this is a sharp distribution it can be interpreted in terms of a tunneling time equal to zero: The velocity of tunneling is infinitely large. In the context of the present work this behaviour is in no conflict with the requirement that no information be transmittable faster than light. The reason is that the WW-approximation limits the validity of our calculation to $t_1, t_2 > z$, i.e. to the inside of the light cone L of the source state preparation event, which was the last occasion when the experimentalist had access to the source (see Fig.2). When the lifetime τ of the source atom is much larger than z/c (which may

be of the order of several nanoseconds) the detection events fall predominantly within L , permitting thereby no faster than light information transfer. It is this region which is covered by our calculation. An improved treatment valid for $\tau \leq z/c$ too (i.e. in both the vicinity of L and outside it) would certainly be of great interest¹.

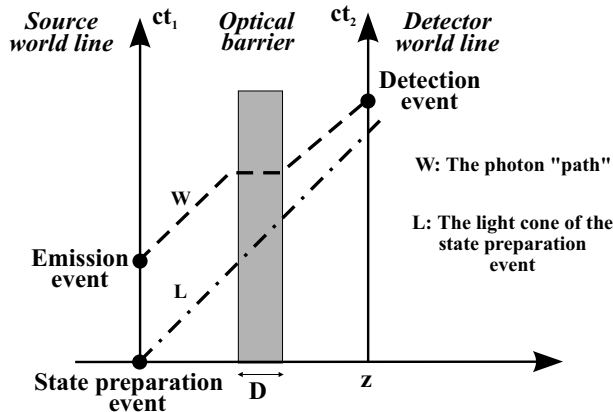


Figure 2. Space-time diagram of the photon path

For a real barrier — or even for our model barrier in a better approximation — the tunneling velocity will probably have a finite value which is greater than the velocity of light in vacuo. From the point of view of relativity theory, however, the point of demarcation is at c . Hence, the conclusions drawn from Figure 2 remain practically unchanged for any tunneling velocity larger than c .

8 FINAL REMARKS

Tunneling time measurements are of two very different kinds: Stationary measurements in which no moments of time are identified at all and experiments in which moments of time of certain real events are determined by using some kind of clocks.

The Berkeley-experiment discussed in Sec.1 is an ingenious example of the first type. No experiment of the second type has so far been performed since it would require precise measurement of extremely small time intervals. Even the purely theoretical analysis of this latter kind of experiments presents a challenge. The present work is an attempt to predict the result of such an experiment. Though the calculation performed is based on a version of the counting rate formulas widely used in quantum optics it cannot be considered

¹The limit $\tau \rightarrow 0$ would be of special importance since it is closely related to the situation when the photon is released at the moment of pressing a "release button" by the experimentalist in any freely chosen instant of time. When $\tau \leq z/c$ the precise nature of the state preparation event requires also closer examination.

completely satisfactory [11]. Quantum theory provides unambiguous rules for the calculation of the probability $p(t_1, t_2)$ from which the correlation function $w(t_1, t_2)$ is obtained by differentiation. Though the rules of quantum theory ensure the positivity of $p(t_1, t_2)$ they don't render it a nondecreasing function of its arguments and so the procedure may end with a negative probability density.

The origin of this "positivity problem" may be traced back to the replacement of the spontaneous state reduction of the detectors — a process which falls outside the scope of the Schrödinger equation — by a "naive reduction hypothesis", consisting in the identification of the statement "*The detector has clicked*" with the statement "*The detector atom is in one of its excited states*". This replacement, however, may be accepted only in the limit of weak coupling between the field and the detector when the latter probability is always a nondecreasing function of time. In the general case the rules of quantum theory do not exclude the possibility that this probability decreases in some intervals of time, while the very notion of the "detector" is irreconcilable with such behaviour: For a detector the probability of being excited must never decrease.

A possible solution of the positivity problem would be to take into account in the dynamics of the detector atom the influence of the equipment which it is built into. This would result in introducing some element of irreversibility into the detector's behaviour which might lead to a never decreasing excitation probability. Theories with spontaneous reduction ([15], [16]) might be also of significance in this respect. Since to first order in the detector-field interaction no positivity problem arises, it may, perhaps, be reasonably expected that in the weak coupling limit the future detector theory will be essentially reduced to our atomic detectors treated in first order perturbation theory on the basis of the naive reduction hypothesis.

We may hence conclude that an experiment of the second kind might well contradict the theory in its present state even if the calculations themselves are irreproachable, reflecting thereby our insufficient knowledge of quantum physics and, perhaps, suggesting the direction toward its completion.

The situation with the experiments of the first kind is quite the opposite. They belong to the domain of phenomena where the applicability of quantum theory has already been abundantly demonstrated. Therefore, their outcome can in principle be calculated in advance and the corresponding time parameter be deduced from this calculation: no contradiction with known principles is expected. In particular, superluminal tunneling under such stationary circumstances never contradicts special relativity since the tunneling process is not accompanied by flow of information, referring to moments of time. In an experiment with clocks, on the other hand, superluminal tunneling would in general contradict relativity theory. Since the theoretical analysis of this experiment performed in the present work does not exclude completely the possibility of superluminal information transfer (see the end of the previous section) the situation deserves careful consideration.

Acknowledgement: The author is deeply indebted to Gábor Hráskó without whose inspiring curiosity this research would not have been pursued.

A DERIVATION OF THE COUNTING RATE FORMULA

In this Appendix the derivation of the counting rate formula (2), (3) based on the lecture [11] is outlined.

Consider the system, consisting of the source atom, the photon field and the atomic photon detector. The detector which signalizes the moment of the photon emission need not be considered explicitly. The total Hamiltonian of this system is $\mathcal{H} = H_s + H_f + H_{sf} + H_d + H_{df}$ where H_s , H_f , H_d are the Hamiltonians of the source, the field and the detector respectively while H_{sf} , H_{df} are the corresponding interactions.

In order to incorporate into the calculation the irreversible nature of the observations of the source and the photon detector at the moments t_1 and t_2 we assume that at these moments the corresponding interactions H_{sf} and H_{df} are switched off. This assumption will be referred to as the "irreversibility hypothesis".

The ground state and the excited states of the detector will be labelled by γ and ϵ . The initial state of the system is

$$|0\rangle = |0\rangle \otimes |\gamma\rangle \equiv |e, vac, \gamma\rangle.$$

Let us work in the Heisenberg picture (labelled by the superscript h) in which the dynamical quantities are driven by \mathcal{H} . After the moment of the first observation the state of the system becomes

$$|intermediate\rangle = \begin{cases} P_g^h(t_1)|0\rangle & \text{if } t_1 < t_2, \\ P_\epsilon^h(t_2)|0\rangle & \text{if } t_2 < t_1. \end{cases}$$

Here P_g and P_ϵ are projectors on the ground state of the source and the excited state ϵ of the detector.

After the second observation the state becomes

$$|t_1, t_2, \epsilon\rangle = \begin{cases} P_\epsilon^h(t_2)P_g^h(t_1)|0\rangle & \text{if } t_1 < t_2, \\ P_g^h(t_1)P_\epsilon^h(t_2)|0\rangle & \text{if } t_2 < t_1, \end{cases}$$

wich can also be written as

$$|t_1, t_2, \epsilon\rangle = T(P_\epsilon^h(t_2)P_g^h(t_1))|0\rangle,$$

where T denotes time-ordering.

After having performed the observations the source is in its ground state, the detector is in one of its excited states and no photon is present. The state $|t_1, t_2, \epsilon\rangle$ is, therefore, identical to $|g, vac, \epsilon\rangle$ except that its norm is smaller than unity. The probability $p(t_1, t_2)$ introduced in Sec.2 is equal to the square of this norm summed over ϵ :

$$p(t_1, t_2) = \sum_{\epsilon} |(g, vac, \epsilon|T(P_\epsilon^h(t_2)P_g^h(t_1))|0\rangle|^2. \quad (43)$$

Our aim now is to eliminate from this formula the explicit reference to the photon detector (except its spectral sensitivity).

Introduce the interaction picture labelled by i by means of the unitary operator

$$V(t, 0) = e^{i\mathcal{H}_0^s t} \cdot e^{-i\mathcal{H}^s t},$$

where s indicates Schrödinger-picture and $\mathcal{H}_0^s = \mathcal{H}^s - H_{df}^s$. In this picture the development of the states is governed by $H_{df}^i(t) = V(t, 0)H_{df}^h(t)V^+(t, 0)$:

$$i\dot{V}(t, 0) = H_{df}^i(t)V(t, 0),$$

the solution of which to first order in the detector-field interaction is

$$V(t, 0) = 1 - i \int_0^t d\tau \cdot H_{df}^i(\tau). \quad (44)$$

Since $\dot{\mathcal{P}}_\epsilon^i = i[\mathcal{H}_0^s, \mathcal{P}_\epsilon^i] = 0$ we have $\mathcal{P}_\epsilon^i(t) = \mathcal{P}_\epsilon^s$ and hence

$$T(\mathcal{P}_\epsilon^h(t_2)P_g^h(t_1)) = \begin{cases} V^+(t_2, 0)\mathcal{P}_\epsilon^s V(t_2, t_1)P_g^i(t_1)V(t_1, 0) & \text{if } t_1 < t_2, \\ V^+(t_1, 0)P_g^i(t_1)V(t_1, t_2)\mathcal{P}_\epsilon^s V(t_2, 0) & \text{if } t_2 < t_1. \end{cases}$$

These expressions are to be calculated to first order, using (44).

The first line ($t_1 < t_2$) gives

$$\begin{aligned} & V^+(t_2, 0)\mathcal{P}_\epsilon^s P_g^i(t_1) - \\ & -iV^+(t_2, 0)\mathcal{P}_\epsilon^s \left[\int_{t_1}^{t_2} dt \cdot H_{df}^i(t)P_g^i(t_1) + \int_0^{t_1} dt \cdot P_g^i(t_1)H_{df}^i(t) \right] = \\ & = V^+(t_2, 0)\mathcal{P}_\epsilon^s P_g^i(t_1) - iV^+(t_2, 0)\mathcal{P}_\epsilon^s \int_0^{t_2} dt \cdot T(H_{df}^i(t)P_g^i(t_1)). \end{aligned}$$

For the second line ($t_2 < t_1$) we have analogously

$$\begin{aligned} & V^+(t_1, 0)P_g^i(t_1)\mathcal{P}_\epsilon^s - \\ & -iV^+(t_1, 0) \left[\int_{t_2}^{t_1} dt \cdot P_g^i(t_1)H_{df}^i(t)\mathcal{P}_\epsilon^s + \int_0^{t_2} dt \cdot P_g^i(t_1)\mathcal{P}_\epsilon^s H_{df}^i(t) \right]. \end{aligned}$$

In the first integral of the last line $t > t_2$. Therefore, by the irreversibility hypothesis we have $H_{df}^i(t) = 0$ in it. Moreover, in the remaining term \mathcal{P}_ϵ^s can be brought in front of the integral since $P_g^i(t_1)$ does not depend on H_{df}^s . We have, therefore,

$$T(\mathcal{P}_\epsilon^h(t_2)P_g^h(t_1)) = C(t_2, t_1) - iV^+(t_1, 0)\mathcal{P}_\epsilon^s \int_0^{t_2} dt \cdot T(H_{df}^i(t)P_g^i(t_1)), \quad (45)$$

where

$$C(t_2, t_1) = \begin{cases} V^+(t_2, 0)P_g^i(t_1)\mathcal{P}_\epsilon^s & \text{if } t_1 < t_2, \\ V^+(t_1, 0)P_g^i(t_1)\mathcal{P}_\epsilon^s & \text{if } t_2 < t_1. \end{cases}$$

When (45) is substituted into (43) the term $C(t_2, t_1)$ gives no contribution since $\mathcal{P}_\epsilon^s|0\rangle = 0$:

$$p(t_1, t_2) = \sum_\epsilon \left| \langle g, vac, \epsilon | V^+(t_1, 0) \mathcal{P}_\epsilon^s \int_0^{t_2} dt \cdot T(H_{df}^i(t) P_g^i(t_1)) | 0 \rangle \right|^2.$$

Now, to first order in H_{df} the operator V^+ must be replaced by unity and since $\langle g, vac, \epsilon | \mathcal{P}_\epsilon^s = \langle g, vac, \epsilon |$, we have

$$p(t_1, t_2) = \sum_\epsilon \left| \langle g, vac, \epsilon | \int_0^{t_2} dt \cdot T(H_{df}^i(t) P_g^i(t_1)) | 0 \rangle \right|^2.$$

Assume now that $H_{df} = q \cdot \varphi$ where q acts in the Hilbert-space of the photon detector. For an arbitrary dynamical quantity \mathcal{O} the i -picture and the Schrödinger-picture are connected by the relation

$$\mathcal{O}^i(t) = e^{i\mathcal{H}_0^s t} e^{-i\mathcal{H}^s t} \mathcal{O}^h(t) e^{i\mathcal{H}^s t} e^{-i\mathcal{H}_0^s t} = e^{i\mathcal{H}_0^s t} \mathcal{O}^s e^{-i\mathcal{H}_0^s t}. \quad (46)$$

The Hamiltonian \mathcal{H}_0 is the sum of H_d and the Hamiltonian of the source-field system ($H_s + H_f + H_{sf}$) which commute with each other. Hence for q equation (46) reduces to

$$q^i(t) = e^{iH_d^s t} q e^{-iH_d^s t},$$

and for such operators as φ and P_g which are independent of the photon detector it gives $\mathcal{O}^i = \mathcal{O}^H$ where the superscript H refers to the Heisenberg-picture introduced in Sec.2.

Assuming, that $H_d^s|\gamma\rangle = 0$ and $H_d^s|\epsilon\rangle = \omega_\epsilon|\epsilon\rangle$ we have

$$p(t_1, t_2) = \sum_\epsilon |\langle \epsilon | q | \gamma \rangle|^2 \cdot \left| \langle g, vac | \int_0^{t_2} dt \cdot e^{i\omega_\epsilon t} T(\varphi^H(t, \vec{r}) P_g^H(t_1)) | 0, vac \rangle \right|^2. \quad (47)$$

Since the spectral sensitivity is given by the relation

$$\tilde{\sigma}(\omega) = \sum_\epsilon \delta(\omega - \omega_\epsilon) |\langle \epsilon | q | \gamma \rangle|^2,$$

(47) becomes identical to the working formulae given in Sec.2.

B DERIVATION OF THE FORMULA (24)

Consider the solution $v_+(\omega_z|z)$ of the equation (9) given in (10). Since the equation is a real linear one, the combination

$$\begin{aligned} & v_+^*(\omega_z|z) - R^* v_+(\omega_z|z) = \\ & = T^* e^{i\omega_z(a-b)} \begin{cases} \frac{1-|R|^2}{T^*} e^{-i\omega_z(z-t)} & \text{if } z < a, \\ e^{-i\omega_z(z-b)} - \frac{R^* T}{T^*} e^{2i\omega_z(b-a)} e^{i\omega_z(z-b)} & \text{if } z > b. \end{cases} \end{aligned}$$

is also a solution which contains an incoming wave from the right. Comparing this solution with (11) we have

$$\begin{aligned} T' &= \frac{1 - |R|^2}{T^*} \\ R' &= -R^* \frac{T}{T^*} e^{2i\omega_z(b-a)}. \end{aligned}$$

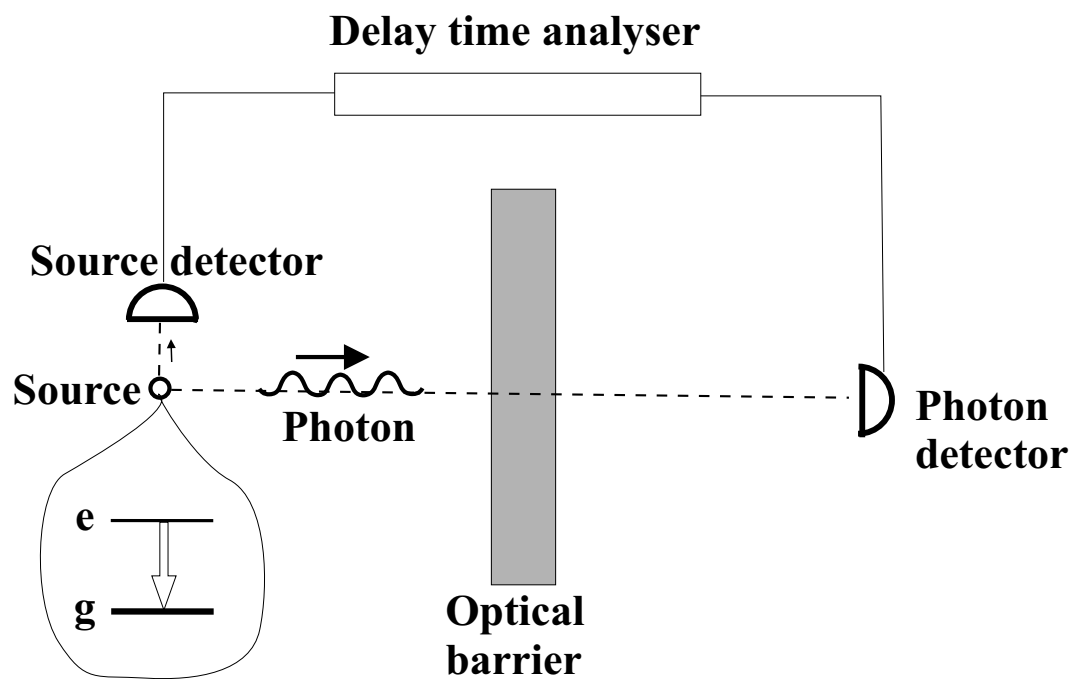
The first of these equations combined with the conservation of the probability $|T|^2 + |R|^2 = 1$ gives $T' = T$ while the second one can be rewritten in the form

$$TR^* e^{-2i\omega_z a} + T^* R' e^{-2i\omega_z b} = 0 \quad (48)$$

which is used in Sec.5.

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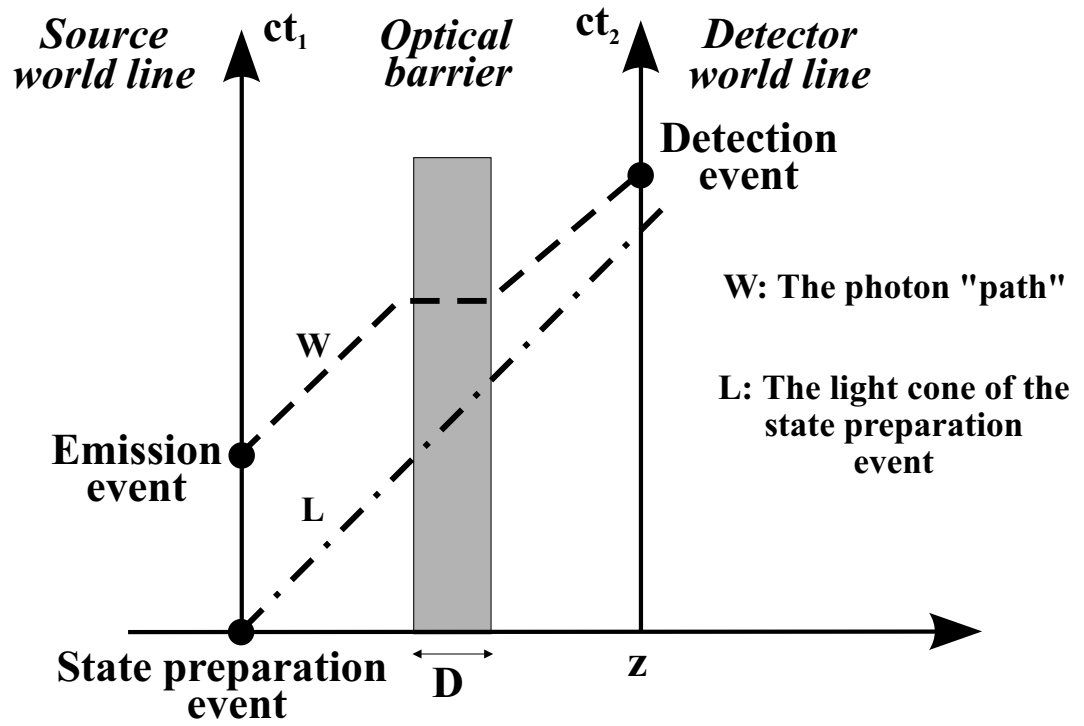


Figure captions

- Figure 1. The setup for the time correlation measurement
Figure 2. Space-time diagram of the photon path